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Optimization of Flexible Multibody Dynamic Systems Using the Equivalent Static Load Method

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Recently, an algorithm for dynamic response optimization transforming dynamic loads into equivalent static loads has been proposed. In later research, it was proved that the solution obtained by the algorithm satisfies the Karush–Kuhn–Tucker necessary conditions. In the present research, the proposed algorithm is applied to the optimization of flexible multibody dynamic systems. The equivalent static load is derived from the equations of motion for a flexible multibody dynamic system. The equivalent load is utilized in sequential static response optimization of the flexible mutibody dynamic system. In the end, the converged solution of the sequential static response optimization is the solution of the original dynamic response optimization. Some standard examples are solved to show the feasibility and efficiency of the proposed method. The control arm of an automobile suspension system is optimized as a practical problem. The results are discussed regarding the application of the proposed algorithm to flexible multibody dynamic systems.

Nomenclature

b	=	design variable vector
b_p	=	design variable vector at the pth cycle
$b_{p,k}$	=	kth element of the design variable vector at the pth cycle
$b_{p,\mathrm{kL}}$	=	lower bound of the kth element of the design variable vector at the pth cycle
$b_{p,\mathrm{kU}}$	=	upper bound of the kth element of the design variable vector at the pth cycle
f_{eq}^{u}	=	equivalent static load vector at the uth time grid point
$f_{\mathrm{eq}}^{u}(p)$	=	equivalent static load vector at the uth time grid point at the pth cycle
g_{ju}	=	jth constraint at the uth time grid point
K	=	stiffness matrix
M	=	mass matrix
m	=	number of inequality constraints
q	=	number of time grid points
$r(t)$	=	applied dynamic load vector at time t
$r(t_u)$	=	applied dynamic load vector at the uth time grid point
t	=	time
t_u	=	uth time grid point
$y(t)$	=	displacement vector induced by dynamic load at time t
y_u	=	displacement vector induced by dynamic load at the uth time grid point
$\dot{y}_u$	=	velocity vector induced by dynamic load at the uth time grid point
$\ddot{y}(t)$	=	acceleration vector induced by dynamic load at time t
$\ddot{y}_u$	=	acceleration vector induced by dynamic load at the uth time grid point
z_u	=	displacement vector induced by equivalent static load at the uth time grid point
φ	=	cost function

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Introduction

Motivation

STRUCTURAL optimization is well developed in static response optimization, where the applied loads are static loads, not dynamic loads.[1,2] Although real loads applied to structures are dynamic, most practical applications assume that the applied loads are static. The assumption is originated from the difficulties in dynamic response optimization. One of the difficulties is design sensitivity analysis, which is inevitable in gradient-based optimization methodology. Another difficulty is treating the dynamic constraints.

To evaluate design sensitivity in dynamic response optimization, we have to solve as many differential equations either as the number of design variables or as the number of active constraints.[3,4] If a transformation method such as the augmented Lagrangian method (ALM) is used with the adjoint variable method (AVM), the cost in calculating design sensitivity can be reduced. We have to solve a terminal value problem to acquire the adjoint variable for the augmented functional. If the terminal value has some error that has been accumulated over the entire time interval where the equations of motion are integrated, the adjoint variable for the augmented functional can diverge or be inaccurate. Another drawback of the ALM in dynamic response optimization is that some multiplier updating rules require the gradient of an individual response. These drawbacks reduce the attraction of the usage of ALM and AVM, although computational effort for design sensitivity is considerably reduced.

Another predicament we often encounter in structural dynamic response optimization is how to treat dynamic constraints. To remove the time parameter from the dynamic constraints, it is customary to replace the dynamic constraint with an equivalent integral form or a lot of pointwise constraints.[3,4] In general, this replacement causes the increment of the number of constraints or numerical instability in the optimization procedure. We need extra effort to reduce the number of constraints or to stabilize the optimization procedure.

There are many optimization examples of large structures that are subject to static loads. The existence of many practical examples means that the static response optimization methodology is well established. Design sensitivity evaluation in static response optimization does not need to solve differential equations but needs to solve simultaneous algebraic equations. In the modern computing environment, it is relatively easy to solve large-scale algebraic equations. If the advantage of static response optimization can be exploited in dynamic response optimization, it will be much easier to perform dynamic response optimization of the structure. Basically the objective of the treatment of a dynamic constraint is elimination of the time parameter. After eliminating the time parameter, we try to find the optimum design by using various optimization techniques. At this point, if we have some compensation for the errors of design sensitivity and response, static response optimization can be adopted in dynamic response optimization. This is the basic idea of this study.

Recently, a structural dynamic response optimization procedure using static response optimization technique has been demonstrated.[5,6] The main idea is transformation of a dynamic load into equivalent static loads and subsequent static response optimization with the transformed equivalent static load set. Theoretical background for the method has been provided in subsequent research.[7]

The objective of this paper is to apply the proposed method to optimization of multibody dynamic systems. The studies on dynamic response optimization of multibody dynamic systems have mainly focused on kinematic properties of rigid-body systems. Most examples of the studies handled a few degrees of freedom. Thus, the number of response variables were small, and design sensitivity analyses were not expensive. However, when members in multibody dynamic systems are considered as flexible bodies, we are destined to confront the difficulties in dynamic response optimization as stated above. Because the nature of dynamics in multibody dynamic systems is not different from that of usual structural dynamics, the method proposed in Ref. 6 is expected to handle the optimization of flexible bodies in multibody dynamic systems.

Review of Literatures

Erdman et al.[8] developed a general kineto-elasto-dynamic analysis and synthesis. They used the finite element method (FEM) for the deflection of a link. They regarded the moving links as a series of instantaneous structures at each position. The system force for the external load in the FEM model was derived from rigid-body acceleration. To update the design variable, they did not use a gradient-based line search, but used a trial-and-error method.

Imam and Sandor[9] minimized the mass of links with stress and deviation constraints using the linear programming method. They used the worst-case approach to treat time-dependent constraints. The number of critical points was single, and the critical point occurred at a fixed time in the entire optimization process. In their later research,[10] the optimization problem was formulated as the sequential unconstrained minimization technique. The unconstrained function was minimized by the sequential-linear-programming method. They discussed two types of pointwise constraints to treat the time-dependent constraint. One was similar to the worst-case approach, and the other was similar to the conventional approach. Khan et al.[11] and Thornton et al.[12] minimized the mass of links based on optimality criterion. From the optimality criterion and fully stressed member concept, they derived an explicit rule for the update of the design variable. Sensitivity information that is needed in the updating rule was evaluated by the direct differentiation method (DDM). Dynamic constraints were replaced by a single worst-case constraint. But they pointed out that the critical response value does not always occur at the same time in the optimization process.

Cleghorn et al.[13] modified Khan's method.[11,12] First, they found an initial design by using Khan's method with rigid-body stress analysis. Starting from this initial design, they applied Khan's method once again with finite element stress analysis to find the final design. Zhang and Grandin[14] modified Khan's method and used Cleghorn's modification.[13] Xianmin et al.[15] considered frequency constraint based on instantaneous structure concept. Yu and Smith[16] discussed the effects of change of the cross-sectional parameters of links in the dynamics of elastic mechanisms. Dias and Pereira[17] performed analytical sensitivity analysis of rigid-flexible multibody systems using DDM. As conventional in the analysis of flexible multibody dynamics, they applied the mode superposition method to solve the dynamic equation. Oral and Ider[18] minimized the mass of flexible robot arms with stress and deviation constraints. Worst-case approach was used to treat dynamic constraints. They showed that pointwise constraint treatment is superior to a transformed equivalent constraint such as the Kreisselmeier–Steinhauser function.[19] Etman et al.[20] adopted the approximation concepts from structural optimization in the optimization of flexible multibody dynamic systems. They used linear approximation of response using intermediate variables. And they took advantage of the combination of constraint screening strategy and pointwise constraints in treating dynamic constraints.

Most studies in this area have dealt with very small-scale problems such as four-bar linkage or two-bar robot arm. The scarcity of a large-scale example seems to be caused by the difficulties in dynamic response optimization as briefly stated in the preceding subsection. Any breakthrough in the dynamic structural optimization field would help us to handle large-scale flexible multibody dynamic systems.

Equivalent Static Load

Definition of Equivalent Static Load

The term "equivalent static load" has been used mainly to replace the effect of the dynamic load. The effect to be replaced by the equivalent static load might not be limited to a single case. One of the effects to be replaced can be displacement induced by the dynamic load. That is, one of the purposes of introducing equivalent static load is to generate a displacement field that is similar to that induced by the dynamic load.

The concept of equivalent static load has been widely used in civil engineering. In most cases of civil engineering, the equivalent static load is used to predict the displacement of a point of interest such as the middle point of the span of a bridge. As a result, most equivalent static loads in the study have a tendency to lose information regarding time. To obtain the equivalent static load, experimental data, amplification factors, and various factors from experience are often used.[21–24] Furthermore, a definition of equivalent static load is not clearly defined. Therefore, it is necessary to clarify the definition of equivalent static load in this study. It is defined as follows.

Definition 1: When a dynamic load is applied to a structure, the equivalent static load is defined as the static load that makes the same displacement field as that by a dynamic load at an arbitrary time.

Starting from definition 1, we can derive the equivalent static load using the finite element method. The equations of the motion of a structure under a dynamic load can be written as

$$M(b)\ddot{y}(t) + K(b)y(t) = r(t) \tag{1}$$

where the damping effect is ignored. Rearranging Eq. (1) yields

$$K(b)y(t) = r(t) - M(b)\ddot{y}(t) \tag{2}$$

or

$$K(b)y(t) = f_{\text{eq}} \tag{3}$$

where

$$f_{\text{eq}} = r(t) - M(b)\ddot{y}(t) \tag{4}$$

According to definition 1, f_{eq} in Eq. (3) is the equivalent static load at time t.

Note that the equivalent static load, from Eq. (4), can be made from the external load and inertia force. Thus, the equivalent static load is an implicit function of design variables. And even though the external force is applied to a single point of a structure, the equivalent static load is applied to all degrees of freedom of the structure. This is clear because the elements of the vector of the inertia force, $M(b)\ddot{y}(t)$, in the equivalent static load f_{eq} in Eq. (4) generally have nonzero values.

From Eq. (4), the equivalent static load can only be obtained after performing transient analysis of the structure. That is, we are trying to calculate known displacement fields using the equivalent static load. From the analysis viewpoint, the equivalent static load seems to be useless. However, the goal of the equivalent static load in the present study is not to predict displacement induced by a dynamic load but to regenerate the known displacement during an optimization procedure, which will be presented in this paper. In other words, the equivalent static load in the present study is not an analysis-oriented load but a design-oriented load. The following subsection shows the usage of the design-oriented equivalent static load in dynamic response optimization.

Similarly, the equivalent inertia load can be defined as follows:

$$p_{\mathrm{eq}} = M(b)\ddot{y}(t) = r(t) - K(b)y(t) \tag{5}$$

which is defined for regeneration of acceleration in the optimization process.

Dynamic Response Optimization Using the Equivalent Static Load Method

It is common practice to handle Eq. (1) in a discrete time domain in the practical approach. Thus, the equivalent static/inertia load can also be calculated in the discrete time domain. The equivalent static/inertia load at uth time grid point f_{eq}^u can be obtained as

$$f_{\mathrm{eq}}^u = K(b)y_u = r(t_u) - M(b)\ddot{y}_u, \qquad u = 1, \ldots, q \tag{6}$$

$$p_{\mathrm{eq}}^u = M(b)\ddot{y}_u = r(t_u) - K(b)y_u, \qquad u = 1, \ldots, q \tag{7}$$

From Eq. (6), the number of the equivalent static load vectors is q.

Consider the following dynamic response optimization problem in a discrete time domain.

$$\min \qquad \varphi(b) \tag{8a}$$

$$\text{s.t.} \qquad M(b)\ddot{y}_u + K(b)y_u = r(t_u), \qquad u = 1, \ldots, q \tag{8b}$$

$$g_{\mathrm{ju}}(b, y_u, \ddot{y}_u) \le 0, \qquad j = 1, \ldots, m, \quad u = 1, \ldots, q \tag{8c}$$

This formulation is the usual form in structural dynamic response optimization because a velocity-based constraint does not appear in most applications. According to the algorithm proposed in Ref. 6, instead of directly solving Eq. (8), we can repeatedly solve the following static response optimization problem, which uses a set of equivalent static/inertia loads:

$$\min \qquad \varphi(b) \tag{9a}$$

$$\text{s.t.} \qquad K(b)y_u = f_{\mathrm{eq}}^u, \qquad u = 1, \ldots, q \tag{9b}$$

$$M(b)\ddot{y}_u = p_{\mathrm{eq}}^u, \qquad u = 1, \ldots, q \tag{9c}$$

$$g_{\mathrm{ju}}(b, y_u, \ddot{y}_u) \le 0, \qquad j = 1, \ldots, m, \quad u = 1, \ldots, q \tag{9d}$$

The algorithm for dynamic response optimization using the equivalent static/inertia load is as follows:

1) Set $p = 0$, $b_p = b_0$.
2) Perform a transient analysis in Eq. (1) with b_p.
3) Calculate equivalent static/inertia loads in the time domain as

$$f_{\mathrm{eq}}^u = K(b_p)y_u, \qquad u = 1, \ldots, q \tag{10}$$

$$p_{\mathrm{eq}}^u = M(b_p)\ddot{y}_u, \qquad u = 1, \ldots, q \tag{11}$$

4) When $p = 0$, go to step 5. When $p > 0$ and if

$$\sum_{u=1}^{q} \left\| f_{\mathrm{eq}}^u(p) - f_{\mathrm{eq}}^u(p-1) \right\| < \varepsilon \tag{12}$$

stop. Otherwise go to step 5.

5) Solve the following static response optimization problem:

$$\min \qquad \varphi(b_{p+1}) \tag{13a}$$

$$\text{s.t.} \qquad K(b_{p+1})z_u = f_{\mathrm{eq}}^u, \qquad u = 1, \ldots, q \tag{13b}$$

$$M(b_{p+1})a_u = p_{\mathrm{eq}}^u, \qquad u = 1, \ldots, q \tag{13c}$$

$$g_{\mathrm{ju}}(b_{p+1}, z_u, a_u) \le 0, \qquad j = 1, \ldots, m, \quad u = 1, \ldots, q \tag{13d}$$

$$b_{p+1,\mathrm{kL}} \le b_{p+1,k} \le b_{p+1,\mathrm{kU}}, \qquad k = 1, \ldots, n \tag{13e}$$

where z_u and a_u are, respectively, displacement and acceleration generated by equivalent static/inertia load in the static response optimization process.

6) Set $p = p + 1$, and go to step 2.

From step 2 to step 6, one cycle is completed. Because the equivalent static/inertia load is fixed in step 5, displacement and acceleration vectors are approximately updated during the process of Eq. (13). However, the equivalent loads are the functions of the design variables y and $\ddot{y}$ of Eq. (1). Thus, the cycle needs to be repeated in order to update the equivalent static/inertia load as the design variables are changed. In the equivalent static load method, transient analysis is performed once every cycle. In traditional dynamic response optimization, we should perform as many transient analyses as the number of design variables or as the number of active constraints in an iteration. Generally, the total number of transient analyses that is required to obtain a converged solution in the equivalent static load method is far less than that of traditional dynamic response optimization. The examples in this paper show the efficiency of the method. It is proved that the solution obtained by this algorithm is an optimum solution of the original dynamic response optimization problem of Eq. (8) (Ref. 7). Various examples also have been solved with the algorithm.[5,6]

Because the time interval $[0, T]$ is divided into q time grid points, the number of equivalent static load vectors is q. Thus there are multiple loading conditions in the static response optimization formulation. It is not difficult to solve a static response optimization problem under multiple loading conditions. Furthermore, if we apply the constraint screening strategy to the static response optimization process in the equivalent static load method, the actual number of constraints to be considered is drastically reduced.

Optimization of Flexible Multibody Dynamic Systems

Inspired by the demand of better kinematic and dynamic performance of high-speed mechanisms, enormous progress in flexible multibody dynamic systems has been achieved during the past decades.[25–27] However, there are not many studies on the optimization of flexible multibody dynamic systems.

Equations of Motion of Flexible Multibody Dynamic Systems

The equations of motion that govern a flexible multibody dynamic system can be expressed as follows[27]:

$$\begin{bmatrix} m_{\mathrm{RR}}^i & m_{R\theta}^i & m_{\mathrm{Rf}}^i \\ & m_{\theta\theta}^i & m_{\theta f}^i \\ \text{symmetric} & & m_{\mathrm{ff}}^i \end{bmatrix} \begin{bmatrix} \ddot{R}^i \\ \ddot{\theta}^i \\ \ddot{q}_f^i \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & K_{\mathrm{ff}}^i \end{bmatrix} \begin{bmatrix} R^i \\ \theta^i \\ q_f^i \end{bmatrix} = -\begin{bmatrix} C_{R^i}^T \\ C_{\theta^i}^T \\ C_{q_f^i}^T \end{bmatrix} \lambda + \begin{bmatrix} (Q_e^i)_R \\ (Q_e^i)_\theta \\ (Q_e^i)_f \end{bmatrix} + \begin{bmatrix} (Q_v^i)_R \\ (Q_v^i)_\theta \\ (Q_v^i)_f \end{bmatrix}$$

$$i = 1, 2, \ldots, n_b \tag{14}$$

where q_f is the nodal deformation; R^i is a set of Cartesian coordinates that defines the location of origin of the floating body reference frame; and θ^i is a set of rotational coordinates that describe the orientation of the floating body reference frame. The terms on the right-hand side of Eq. (14) are the reaction force from the joint constraint, the external load, and the quadratic velocity vector, which includes the effect of the Coriolis and centrifugal force, respectively.[27] All of these terms are time dependent. Thus, flexible multibody dynamic systems expressed by Eq. (14) are subjected to dynamic loads. The optimization of flexible multibody dynamic systems is dynamic response optimization. The proposed algorithm using an equivalent static load is expected to be applicable to the optimization of flexible multibody dynamic systems.

Equivalent Static Load in Flexible Multibody Dynamic Systems

The equivalent static load was defined in definition 1. From definition 1, we have to solve Eq. (14) to obtain the equivalent static load. And Eq. (14) yields the following three equations:

$$m^i_{\mathrm{RR}}\ddot{R}^i + m^i_{R\theta}\ddot{\theta}^i + m^i_{\mathrm{Rf}}\ddot{q}^i_f = -C^T_{R^i}\lambda + \left(Q^i_e\right)_R + \left(Q^i_v\right)_R \qquad (15)$$

$$m^i_{R\theta}\ddot{R}^i + m^i_{\theta\theta}\ddot{\theta}^i + m^i_{\theta f}\ddot{q}^i_f = -C^T_{\theta^i}\lambda + \left(Q^i_e\right)_\theta + \left(Q^i_v\right)_\theta \qquad (16)$$

$$m^i_{\mathrm{Rf}}\ddot{R}^i + m^i_{f\theta}\ddot{\theta}^i + m^i_{\mathrm{ff}}\ddot{q}^i_f + K^i_{\mathrm{ff}}q^i_f = -C^T_{q^i_f}\lambda + \left(Q^i_e\right)_f + (Q^i_v)_f \qquad (17)$$

Suppose that Eq. (14) has been solved for acceleration and displacement. Then Eq. (17) can be rewritten as

$$K^i_{\mathrm{ff}}q^i_f = -m^i_{\mathrm{Rf}}\ddot{R}^i - m^i_{f\theta}\ddot{\theta}^i - m^i_{\mathrm{ff}}\ddot{q}^i_f - C^T_{q^i_f}\lambda + \left(Q^i_e\right)_f + \left(Q^i_v\right)_f \qquad (18)$$

According to definition 1, the equivalent static load in flexible multibody dynamic systems f_{eq} can be stated as

$$f_{\mathrm{eq}} = K^i_{\mathrm{ff}}q^i_f \qquad (19)$$

or

$$f_{\mathrm{eq}} = -m^i_{\mathrm{Rf}}\ddot{R}^i - m^i_{f\theta}\ddot{\theta}^i - m^i_{\mathrm{ff}}\ddot{q}^i_f - C^T_{q^i_f}\lambda + \left(Q^i_e\right)_f + \left(Q^i_v\right)_f \qquad (20)$$

Note that the first two terms in the right-hand side in Eq. (20) represent the effect of the coupling of a rigid-body motion and elastic deformation. The third term can be considered as the inertia force caused by elastic deformation of the flexible body. The fourth term is the reaction force from the joint constraint. The last term includes the effect of the Coriolis force and the centrifugal force. Using the equivalent static load obtained by Eq. (19) or (20), we can apply the algorithm presented in the preceding subsection to the optimization of flexible multibody dynamic systems.

Algorithm for the Optimization of Flexible Multibody Dynamic Systems

The algorithm for the optimization of flexible multibody dynamic systems is as follows:

1) Set $p = 0$, $b_p = b_0$.

2) Perform transient analysis in Eq. (14) with b_p for p.

3) Calculate equivalent static load sets in the time domain as

$$f^u_{\mathrm{eq}} = K_{\mathrm{ff}}(b_p)y^u_{\mathrm{ff}}, \qquad u = 1, \ldots, q \qquad (21)$$

4) When $p = 0$, go to step 5. When $p > 0$ and if

$$\sum_{u=1}^{q} \left\| f^u_{\mathrm{eq}}(p) - f^u_{\mathrm{eq}}(p-1) \right\| < \varepsilon \qquad (22)$$

stop. Otherwise go to step 5.

5) Solve the following static response optimization problem:

$$\min \quad \varphi(b_{p+1}) \qquad (23a)$$

$$\text{s.t.} \quad K_{\mathrm{ff}}(b_{p+1})z_u = f^u_{\mathrm{eq}}, \qquad u = 1, \ldots, q \qquad (23b)$$

$$g_{\mathrm{ju}}(b_{p+1}, z_u) \le 0, \qquad j = 1, \ldots, m, \quad u = 1, \ldots, q \qquad (23c)$$

$$b_{p+1,\mathrm{kL}} \le b_{p+1,k} \le b_{p+1,\mathrm{kU}}, \qquad k = 1, \ldots, n \qquad (23d)$$

6) Set $p = p + 1$, and go to step 2.

The acceleration-based constraints are not considered in the preceding process. With the algorithm proposed here, several flexible multibody dynamic systems will be optimized in the subsequent sections.

Optimization of Sectional Dimensions of a Two-Link Robot Arm

This example has been solved in Ref. 18. A two-link planar robot arm is shown in Fig. 1. Each link is modeled as a two-element beam whose cross section is hollow. The length of each link is 0.6 m. The design variables are the outer diameters of the elements. The wall thickness of the links is set to be 0.1× outer diameter. Young's modulus is 72 GPa, and density is 2700 kg/m^3. The number of modes needed for flexible multibody dynamic analysis is three. The first two modes are the bending modes, and the third mode is the axial deformation mode. For the analysis, Recurdyn[28] is used. For static response optimization, GENESIS[29] is used. Each link is considered as a fixed-free beam. Revolute joint A in Fig. 1 has a mass of 2 kg, and the end effector E has a mass of 1 kg. The cost function is the total weight of the two links. The prescribed motion of E is given by

$$\Delta X_E = \Delta Y_E = (0.5/T)[t - (T/2\pi)\sin(2\pi t/T)] \qquad \forall t \in [0, 0.66] \qquad (24)$$

where T is set to be 0.5 s. Constraints are given by

$$-75.0 \le \sigma_i \le 75.0 \text{ MPa}, \qquad i = 1, 2, 3, 4 \qquad (25)$$

$$\left(\delta y_a^2 + \delta y_b^2\right)^{0.5} \le 0.001 \text{ m} \qquad (26)$$

where Eq. (25) is the stress constraint and Eq. (26) is the deflection constraint; δy_a and δy_b are vertical deflections at nodes a and b, respectively. Initial values of the design variables are 50.0 mm.

The optimization results are shown in Table 1. The number of inner iterations in Table 1 denotes the total iteration number for solving Eq. (23) during the six cycles. Because the static response optimization tool and multibody dynamic analysis tool in the present study are not the same as those in Ref. 18, it is impossible to compare both results directly. However, the overall trend of the optimum values of the two results shows good agreement.

Table 1 Comparison of optimum cross-sectional dimensions of the two-link robot arm, which shows good agreement with the result of Ref. 18 and efficiency of the equivalent static load method

Compared items	Using the proposed algorithm	Ref. 18
D_{11}, mm	54.45	54.266
D_{12}, mm	38.525	44.150
D_{21}, mm	30.4	37.552
D_{22}, mm	22.7	26.315
Cost, N	13.229	15.719
No. of cycles	6	38
No. of inner iterations	14	——

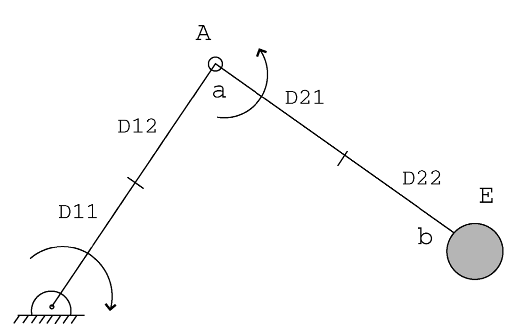


Fig. 1 Two-link robot arm whose links are modeled as finite element beams and whose cross sections are circular and hollow.

Note that the dynamic constraints have been replaced by a single worst-case constraint[3] in Ref. 18, which requires an effort to keep track of the peak point. However, the present practice utilized a plain conventional treatment of dynamic constraints with a constraint screening strategy. The total number of cycles (six cycles) is smaller than that of Ref. 18. This implies that smaller number of transient analyses were performed in the present work because Ref. 18 evaluated the sensitivity information with the overall finite difference method. Furthermore, during the six cycles, the total number of inner iterations for solving Eq. (23) was 14. This small number is as a result of taking advantage of modern static response optimization techniques such as the approximation of response, intermediate design variables, analytical sensitivity information, and so on.

Optimization of Sectional Dimensions of a Four-Bar Mechanism

This example is taken from Ref. 20. A four-bar mechanism shown in Fig. 2 consists of three mobile links and one ground link. Each link is modeled with six beam elements whose sections are circular and solid. Each link is assumed to be a simply supported beam.[30] Young's modulus is 68.95 GPa, and density is 2757 kg/m^3. The length of link 1(l_1), link 2(l_2), link 3(l_3), and link 4(l_4) are 0.3048, 0.9144, 0.7620, and 0.9144 m, respectively. Link 1 rotates at a constant angular velocity ω of 10 π/s. The design variables are diameters of the links. The bending stresses σ are given by

$$\sigma_i^k = \left(4\sqrt{\pi} \big/ A_i\right) M_i^k \tag{27}$$

where A_i is the cross-sectional area of the ith link and M_i^k is the bending moment at node k of the ith link. The cost function is the total mass of the mechanism. The stress constraints are given by

$$\sigma_i^k(b, t) \le 27.58 \text{ MPa}, \qquad \forall t \in [0.3, 0.5] \tag{28}$$

The initial values of the design variables are all 356.8 mm. The first three normal modes are used for the flexible multibody dynamic analysis.

According to Ref. 20, this example has multiple local optimum points. The results of Table 2 can be understood as one of the many local optimum points. Also the difference of analysis tool and optimization tool affected the difference of results of the present work and Ref. 20. The total number of cycles in the present work and Ref. 20 seem to be similar. Reference 20, however, evaluated the sensitivity information using the finite difference method. Thus, the actual number of dynamic analyses in Ref. 20 is at least 45. This is larger than 16, the number of dynamic analyses performed in the present work.

Table 2 Comparison of optimum cross-sectional dimensions of the four-bar mechanism, where Ref. 20 performed 45 transient analyses and the present method performed 16 transient analyses

Compared items	Using the proposed algorithm	Ref. 20
d1, mm	33.5	38.5
d2, mm	23.2	25.2
d3, mm	16.9	20.0
Cost, kg	2.28	2.89
No. of cycles	16	15
No. of inner iterations	22	——

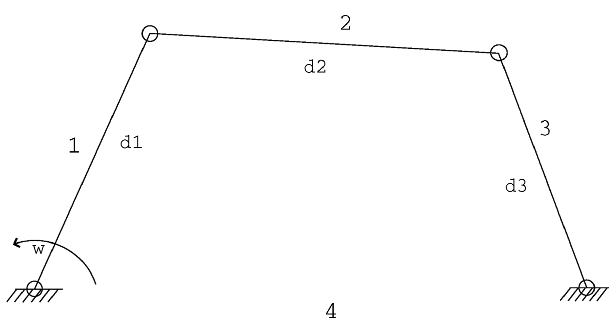


Fig. 2 Four-bar mechanism whose links are modeled as finite element beams and whose cross sections are circular and solid.

Optimization of Sectional Dimensions of a Control Arm in a Suspension System

A control arm in the suspension system shown in Fig. 3 is optimized. The control arm is modeled as a 10-element beam structure. Each beam has an I-shaped cross section as illustrated in Fig. 4b. Each element has four design variables as shown in Fig. 4b. The total number of design variables in the control arm is 40. Young's

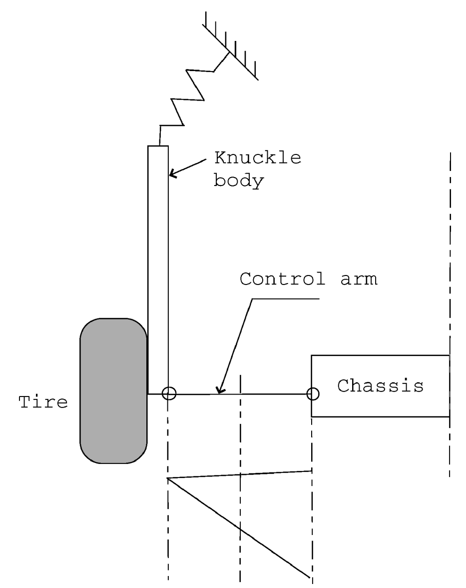
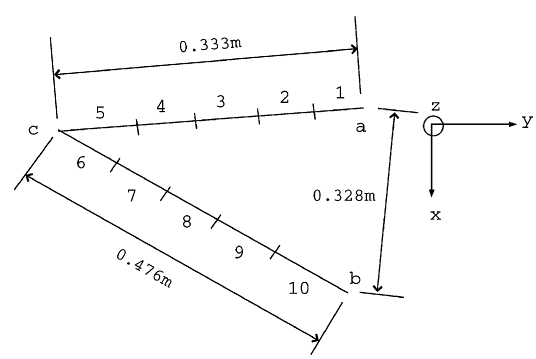
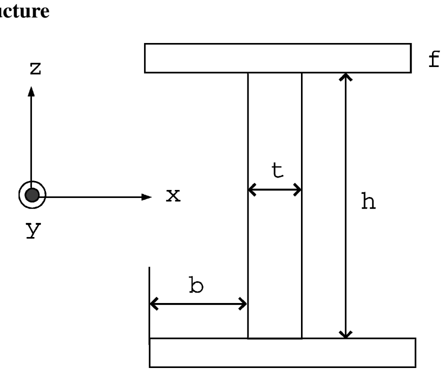


Fig. 3 Suspension system whose lower control arm is modeled with the finite element method.



a) Lower control arm that is modeled as a 10-element beam structure



b) Four design variables of the I-shaped section of each element

Fig. 4 Control arm and its design variables.

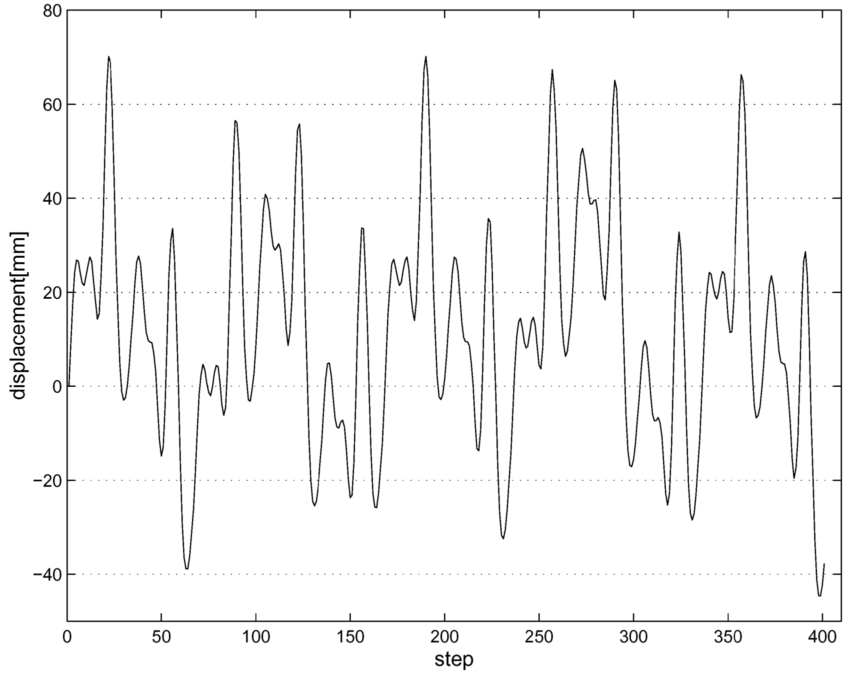


Fig. 5 Prescribed motion of a tire in a suspension system.

Table 3 Optimum cross-sectional dimensions of the control arm that are obtained by the proposed algorithm

Design variable	Optimum value, mm	Design variable	Optimum value, mm
b_1	5.00000	b_6	5.50012
f_1	1.50035	f_6	1.50004
h_1	5.00002	h_6	5.00042
t_1	3.00000	t_6	3.00007
b_2	5.00000	b_7	5.00003
f_2	1.50033	f_7	1.50001
h_2	5.00000	h_7	7.52060
t_2	3.00000	t_7	3.00002
b_3	5.00000	b_8	5.00001
f_3	1.50035	f_8	1.50000
h_3	5.00000	h_8	8.51132
t_3	3.00000	t_8	3.00000
b_4	5.00000	b_9	5.00002
f_4	1.50035	f_9	1.50001
h_4	5.00000	h_9	7.03266
t_4	3.00000	t_9	3.00001
b_5	5.00000	b_{10}	0.71618
f_5	1.50035	f_{10}	1.50000
h_5	5.00000	h_{10}	7.05728
t_5	3.00000	t_{10}	3.00000
Cost, kg	0.1243	——	——

modulus and density are 7.2 GPa and 2700 kg/m^3, respectively. The tire shown in Fig. 3 moves vertically. The prescribed vertical displacement of the tire is shown in Fig. 5. The cost function is the total mass of the control arm. The constraints are given by

$$\sigma \leq 15.0 \text{ MPa} \tag{29}$$

where σ is the maximum stress of the each element. Initial values of b, f, h, and t are 10, 2, 20, and 5 mm, respectively. For the finite element model of the control arm, nodes a and b are assumed to be fixed to the spherical joint. Considered time interval is [0.0, 1.5] s. The time interval is discretized into 400 grid points. Table 3 shows the optimization results.

Note that during the optimization there was a situation where the number of active or violated constraints was approximately 450. Thus, if the traditional approach were used, a lot of effort is needed to evaluate the sensitivity of the constraints. The design variables of element 1 to element 6 have their lower bound values. For elements 7, 8, and 9, the webs are higher than those of the other elements. The length of b–c is longer than that of a–c. Thus, if elements 7, 8, and 9 were not reinforced, deflection by gravity and inertia force would be inevitable.

Discussion of Examples

To calculate the equivalent static load, transient analysis is performed in advance. This is not a disadvantage of the proposed algorithm because the conventional approach also needs to perform transient analysis at the current design point. The additional amount of computation induced by calculating the equivalent static load is only caused by simple postprocessing of the transient analysis result. Because the equivalent static load allows us to evaluate the sensitivity information from linear algebraic equations, the additional amount of computation for calculating the equivalent static load is even ignorable compared with the effort to evaluate the sensitivity information from second-order differential equations.

The lowest natural frequency can be far from its initial value as the design is improved. In that case, the initial coarse time grid can miss actual critical points. Interactive adjustment of the number of time grids is not desirable. Adjustment of the grid causes variation of the number of constraints in the optimization formulation. That is, if once we adjust the time grid during the optimization process, we solve a different optimization problem from the previous optimization problem because the the number of constraints of the two optimization formulations are not the same. In the initial stage, the time grid should be fine enough for detection of the response of the expected final design. But the final design cannot be predicted. Determination of the number of the time grid depends on experience.

Because the finite element method is included in the formulation for flexible multibody dynamic systems, the boundary condition should be imposed on the flexible body. However, determination of the boundary condition between a joint and flexible body has not been clearly addressed. In an initial study on the two-link robot arm example, two types of boundary conditions—a simply supported beam and a fix-free beam—were imposed on the beam for testing the effect of boundary conditions on the analysis result. Analysis results of the two types of boundary conditions showed somewhat different results. The difference could affect the optimization result. Actually, the matter of boundary condition is an ever-present problem in the finite element method. This issue is still under debate.

For transient analysis, the mode superposition method was used to reduce the size of the problem at the reasonable expense of solution accuracy. There are a couple of side issues regarding the solution strategy. One is which modes should be included in terms of computational efficiency. This problem is quite a traditional issue in transient analysis using the mode superposition method. The best way to attack this problem is the direct integration scheme such as the Newmark method.[31] However, there are some restrictions in implementing the Newmark method in multibody dynamics[27,32] because the equations of the motion of multibody dynamic systems are accompanied with kinematic constraints. The other issue is the mode switching problem. This is not a serious problem. Because modal analysis and transient analysis are performed repeatedly, the switched mode can be automatically taken into consideration in the next modal analysis and transient analysis.

Conclusions

The structural dynamic response optimization technique using equivalent static load is applicable to the optimization of flexible multibody dynamic systems. This has been verified by a few examples. Furthermore, efficiency was also improved compared to the previously reported results. Because of the solution strategy of most multibody dynamic system analysis, it is important to select the normal modes of the structure properly.

For further development, one is encouraged to try shape optimization of flexible multibody dynamic systems. Basic discussions for shape optimization already have been done. One of the expected problems is that we are not sure that current multibody dynamics solvers can handle an extremely large structure. During the present research, the solution of the multibody dynamics solver was unstable when a structure with a large number of degrees of freedom and its higher modes were involved. In spite of this instability, however, it is worthy of trying shape optimization of flexible multibody dynamic systems because there has been no attempt to do so.

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